The Kármán line definition

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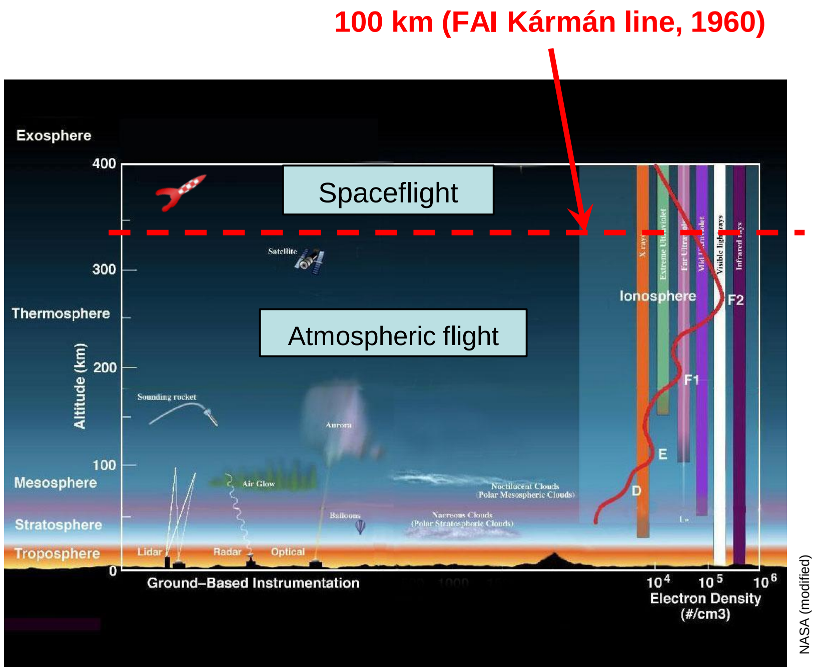
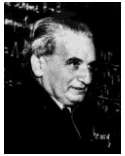
Introduction about the Kármán line



NASA (modified)



NASA

Theodore von Kármán
(1881-1963)

- Kármán line = one convention for defining a "boundary of space" (other exist, e.g. USAF astronaut wings: 50 miles [80.47 km]).
- This concept emerged in space law discussions in the 1950's (name "Kármán line" coined by Andrew G. Haley in a 1957 IAC paper, following discussions with Kármán)
- Set at 100 km by FAI in 1960 (with a group of scientists, including Kármán) for separating aeronautics and astronautics records

FAI definition (1960)

- From “100km Altitude Boundary for Astronautics” (S. Sanz Fernández de Córdoba, FAI website, 2004):

In Aeronautics, level flying higher and higher meant to deal with less and less dense atmosphere, thus to the **need of greater and greater speeds to have the flying machine controllable by aerodynamic forces**. A speed so big in fact, that, above a certain altitude, could be close or even bigger than the circular orbital speed at that altitude (i.e. lift was no longer needed, since **centrifugal force took over; and consequently aerodynamic flight was meaningless**).
Conversely**, in Astronautics, lower and lower orbital flying led to encounter more and more dense atmosphere, so much that it would be impossible to keep the orbit for a number of turns around Earth** without a significant forward thrust (thus making the free fall, or orbiting, concept meaningless). A lot of calculations were made, and finally it was reached the conclusion, accepted by all scientist involved, that around an altitude of 100 Km. the boundary could be set.(...)
It was apparently Von Kármán himself who realized, and proposed to the rest, the very round number of 100 Km (very close to the calculated number).

- This article is the only known source about the origin of the choice of the **100 km** altitude boundary but:
  - It does not contain enough information to understand the calculations behind this choice.
  - Information are sometimes ambiguous or contradictory (different concepts are mixed: aerodynamic controllability, centrifugal force “taking over”, orbit stability and atmospheric drag).

What Kármán himself wrote

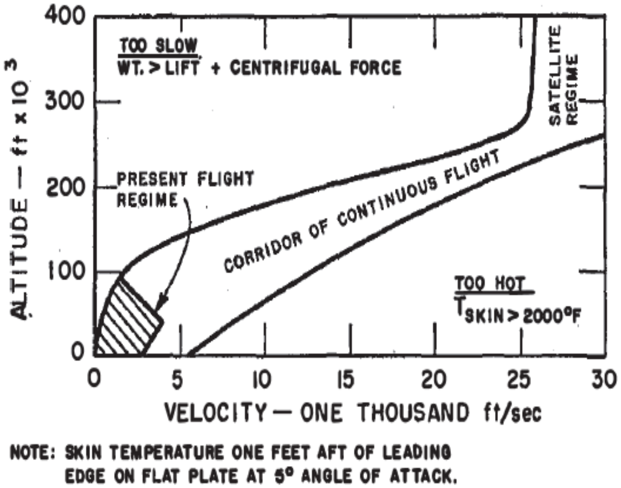
Source: “From Low Speed Aerodynamics to Astronautics”, Pergamon Press, 1963 (conferences by Th. Kármán at University of Maryland, 28-29 April 1961)

“In considering the re-entry problem, we must first ask where is the limit between atmosphere and outer space. **It is a judiciary rather that a scientific problem** (…)
I propose to adopt as a conclusion the graphics on altitude and velocity study for real flight proposed by M. Gazley from “Rand Corporation”. He considered that permanent flight is limited by two extremal conditions:

1/ either the vehicle is to slow to fly because its weight is greater than the sum of lift due to air and centrifugal force

2/ or the vehicle is too hot to fly because the coating’s temperature is greater than the critical temperature of the material. Gazley estimated this critical temperature to 2000° F. He found that permanent flight was limited to a narrow strip on the graphics (…).

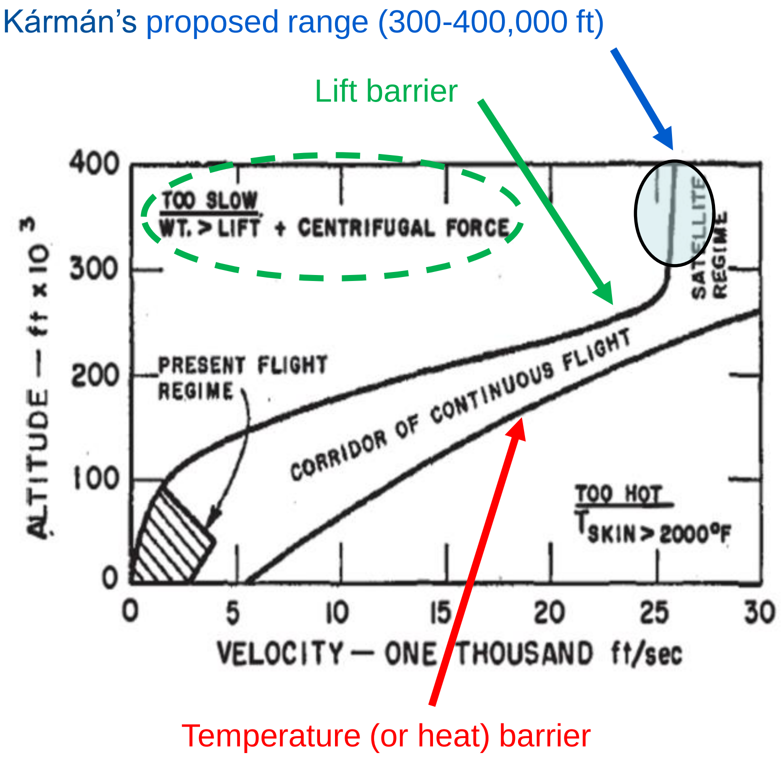
It is evident that when centrifugal force becomes dominant, the vehicle turns into a satellite. That condition can be considered as the beginning of space. **Hence I propose that space starts from 300 or 400,000 ft [91.44 or 121.92 km]”**



Gazley (or “Masson & Gazley”) graphics

(from “Surface Protection And Cooling Systems For High-Speed Flight, IAS National Summer Meeting, Los Angeles, June 18-21, 1956)

Role of the “lift barrier curve” in the Kármán line theory

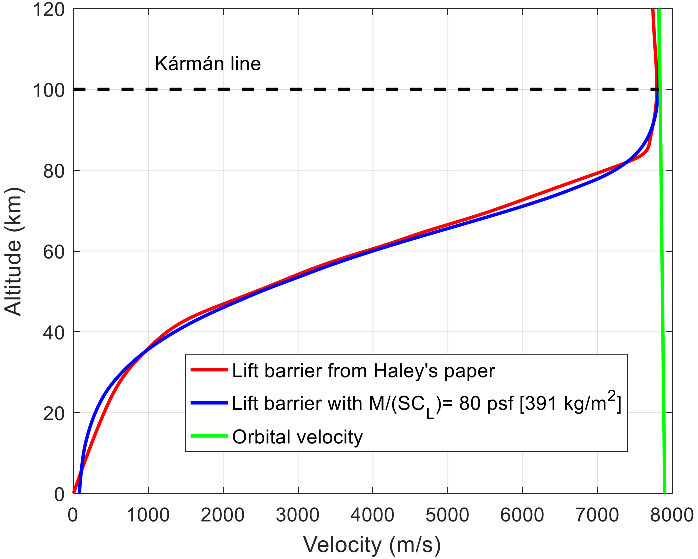


- Kármán apparently never **calculated** precisely an altitude for the boundary of space. He only proposed the approximate range where the “**lift barrier**” curve becomes very steep ([300-400,000 ft] or ~[91-122 km]).

- The (rounded) 100 km altitude eventually set by FAI is definitely consistent with this range.

- **Warning:** the 275,000 ft (83.82 km) is often presented as the “original altitude calculated by Kármán”. It is actually an “**illustrative**” figure provided by Haley in his 1957 IAC paper (a figure that “**may be significantly changed**”). This altitude is actually **outside** Kármán’s proposed range.

Re-creating the "lift barrier" curve



$$V(h) = \sqrt{\frac{\mu_E}{(r_E+h)} - \frac{\rho(h)\cdot(r_E+h)}{2\left(\frac{M}{S\cdot C_L}\right)}}$$

Weight = Lift + Centrifugal force

$$\frac{\mu_E}{(r_E+h)^2} = \frac{\frac{1}{2}\rho(h)V^2}{\left(\frac{M}{S\cdot C_L}\right)} + \left(\frac{V^2}{r_E+h}\right)$$

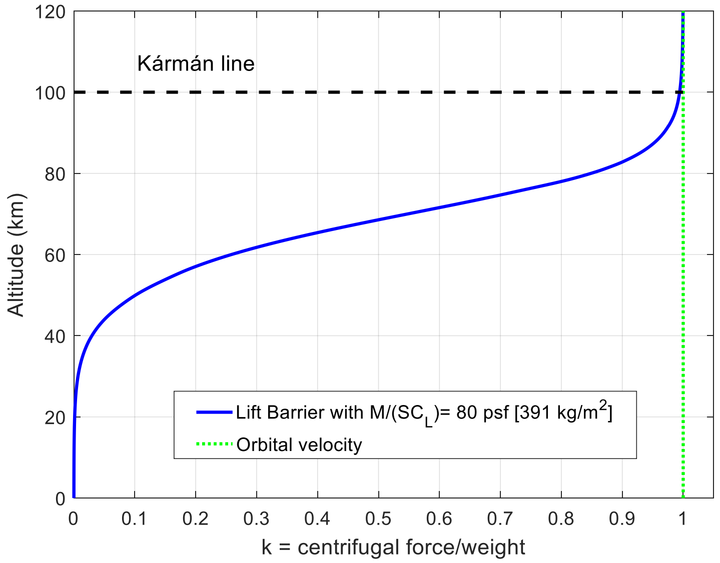
$\rho(h)$	atmos. density (kg/m^3
M	mass (kg).
C_L	lift coefficient (-)
S	aerodyn. surface (m^2)
$\frac{1}{2}\rho(h)V^2$	dyn. pressure (Pa)
M/(S.C$_L$)	lift parameter

$$\text{Lift parameter} = \frac{\text{Wing loading}}{\text{Lift coefficient}} = \frac{\text{Ballistic coefficient}}{\text{Lift to Drag ratio}}$$

$$\frac{M}{S\cdot C_L} = \frac{\left(\frac{M}{S}\right)}{C_L} = \frac{\left(\frac{M}{S\cdot C_D}\right)}{(C_L/C_D)}$$

- The lift barrier curve depends on an hypothesis on M/(S.C$_L$), representing the technical limit.
- In Masson & Gazley's curve: M/(S.C$_L$) ~400 kg/m2 [80 psf]

Physical phenomena represented by the “lift barrier” curve



- Moving up along the curve:
  - Increases the “velocity to orbital velocity” ratio.
  - Increases also the “centrifugal force/weight ratio”, which is related:

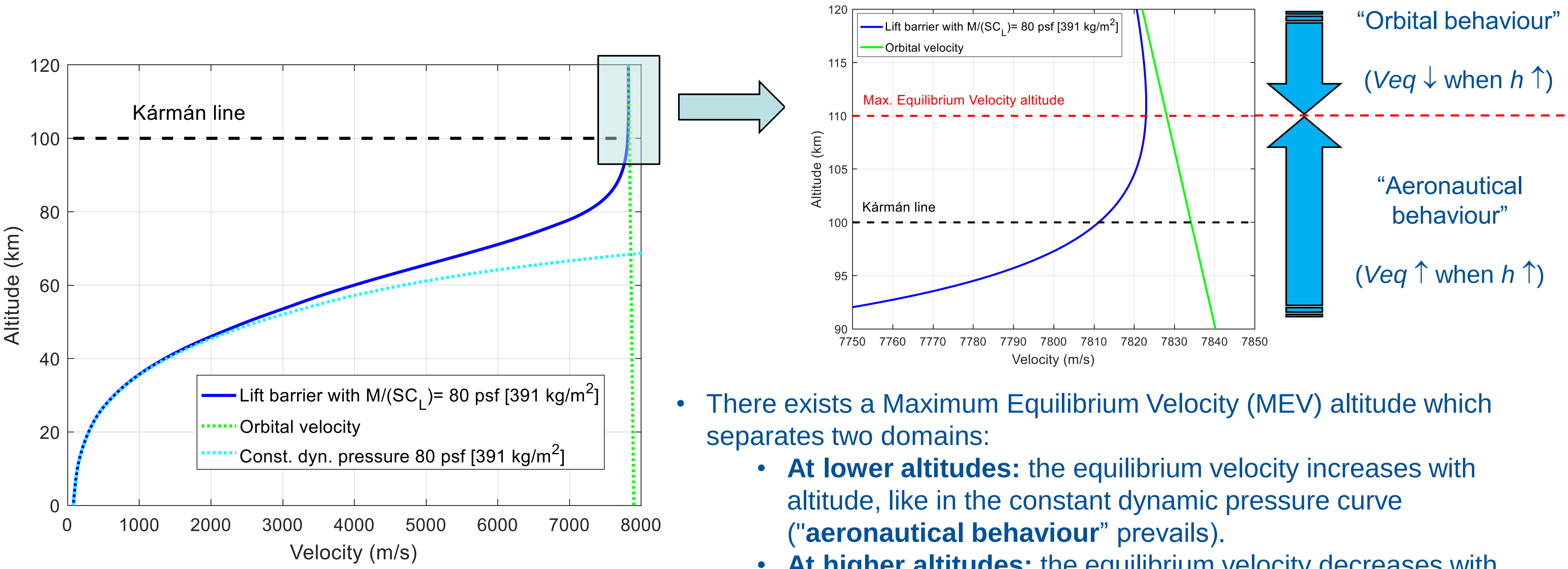
$$\boldsymbol{k} = \frac{\left(\frac{V^2}{r_E + h}\right)}{\left(\frac{\mu_E}{(r_E + h)^2}\right)} = \frac{(r_E + h) \cdot V^2}{\mu_E} = \left[\frac{V}{\sqrt{\frac{\mu_E}{r_{E+h}}}}\right]^{\mathbf{2}}$$

$$k = \frac{\text{Centrifugal force}}{\text{Weight}} = \left[\frac{\text{Velocity}}{\text{Orbital velocity}}\right]^{\mathbf{2}}$$

- Defining a precise boundary based on a threshold on *k* (or $\sqrt{k}$) would be a possibility … but it would be an arbitrary choice (like the rounding to 100 km).

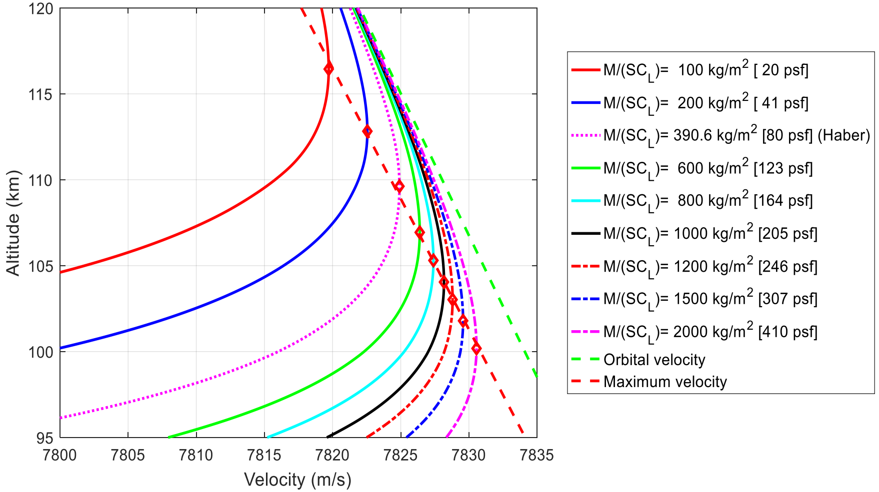
A new proposal to define a *precise* altitude using physics

(IAC-22-C1.IP.8)



- There exists a Maximum Equilibrium Velocity (MEV) altitude which separates two domains:
  - **At lower altitudes:** the equilibrium velocity increases with altitude, like in the constant dynamic pressure curve ("**aeronautical behaviour**" prevails).
  - **At higher altitudes:** the equilibrium velocity decreases with altitude, like in the orbital velocity curve ("**orbital behaviour**" prevails)

A new proposal to define a *precise* altitude using physics



- The Maximum Equilibrium Velocity (MEV) altitude is **~110 km** for $M/(S.C_L)$~400 kg/m2 (original hypothesis in Gazley's curve).

- It is relatively stable wrt. $M/(S.C_L)$:
  - ~112 km for $M/(S.C_L)$ ~200 kg/m2
  - ~105 km for $M/(S.C_L)$ ~800 kg/m2

- With simplifying assumptions (exponential atmospheric density model), we obtain a closed-form expression:

$$h_{MEV} \cong \mathrm{L} \cdot \ln\left(\frac{\rho_0 \cdot r_E{}^2}{2L\left(\frac{M}{S \cdot C_L}\right)}\right) \qquad \rho(h) = \rho_0 \cdot e^{-\frac{h}{L}}$$

- For fun (?): the h_{MEV} altitude boundary is estimated at **113 km for Mars** and **303 km for Venus**.

Conclusion

- The Maximum Equilibrium Velocity (MEV) definition identifies more precisely the physical phenomenon that occurs in the altitude range designated by Kármán. It is definitely compatible with Kármán's guidelines and altitude range (and relatively close to FAI's rounded value: 110 km vs. 100 km current value).

- Unlike the 100 km altitude, it does not rely on an arbitrary rounding, but only on physics. It also relies on the same technical hypothesis (estimation of the minimum possible lift parameter, but the MEV altitude is relatively stable wrt. this parameter).

- The Kármán line theory **is not the only possible definition** relying on aerodynamic forces:
  - Kármán line theory → based on **lift**
  - USAF limit → based on **aerodynamic controllability** (cf. Jenkins, 2005)
  - McDowell's proposal (2018) → based on **drag** ("satellite" point of view)

- The question of the boundary of space is, in any case, **a matter of convention.**

Epilogue

"(…) I am sure that these knotty problems of law will not be solved in my lifetime. In any case I prefer not to speculate in this area but to return to the relative ease and comfort of solving purely scientific questions, or looking into the future of scientific development."

Theodore von Kármán (1881-1963), from his autobiography "The Wind and Beyond"